Particle Image Velocimetry Measurements of an Underexpanded Supersonic Jet

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Abstract

An investigation into the methodology of analyzing a supersonic jet via Particle Image Velocimetry (PIV) using solid seed particles is presented. An isothermal jet at \( M_a = 1.3 \) through a 2.5mm De Laval nozzle is examined. A Lorenz-Mie scattering analysis is used to determine the scalars of the flow, while a Multigrid Cross Correlation Digital Particle Image Velocity (MCDCPIV) approach measures the velocity field. A classical free turbulent jet structure is evident, with a double hump velocity profile characteristic of an underexpanded jet. The effectiveness of the flow seeding approach is also examined, with the specific momentum of the incoming flow shown to be the critical factor in the operation of the seeder.

1. INTRODUCTION

This paper describes a preliminary investigation into the use of Particle Image Velocimetry (PIV) to investigate high speed, high temperature flows. High speed flow and high temperature flow are two separate problems that pose their own distinct challenges. This paper is focused entirely on the issues intrinsic to the analysis of high speed flow. However a velocity (MCCDPIV) approach measures the velocity field. A is shown to be the critical factor in the operation of the seeder. An isothermal jet at \( M_a = 1.3 \) through a 2.5mm De Laval nozzle is examined. A Lorenz-Mie scattering analysis is used to determine the scalars of the flow, while a Multigrid Cross Correlation Digital Particle Image Velocity (MCDCPIV) approach measures the velocity field. A classical free turbulent jet structure is evident, with a double hump velocity profile characteristic of an underexpanded jet. The effectiveness of the flow seeding approach is also examined, with the specific momentum of the incoming flow shown to be the critical factor in the operation of the seeder.

Another limiting factor in terms of the accuracy of high speed PIV is the ability of the seed particle to follow the flow. For high speed flow, particles that will not undergo change due to temperature and pressure are required. Metal oxide particulates fit this requirement, however the higher density of these particulates limit their ability to faithfully follow the flow, and a careful analysis must be undertaken to determine the necessary particle dimensions to map out the structures of interest in the flow.

A tracer particle’s ability to follow the flow can be quantitatively judged on two criteria. The first is the particle’s ability to follow turbulent structures within the flow, and a substantial amount of research has been dedicated to quantifying this, which is covered in detail in Melling [1].

The second criterion is only of relevance in the case of transonic and supersonic flows, that being the response of particles to shockwaves. Shockwaves represent a step decrease in flow velocity, and the seed particle’s response to this step decrease must be quantified. Melling [1] specifies that the standard methodology for quantifying particle response is particle relaxation time, denoted \( \tau_p \). Particle relaxation time is defined as the time after a step change in flow velocity required for the velocity lag \( \| \mathbf{U}_p - \mathbf{U}_f \| \) to be reduced by a factor of \( 1/e = 0.368 \). Melling [3] provides the basic equation from which the particle relaxation time is to be estimated, stating that empirical estimates of the drag coefficient for the particles are required. Urban and Mungal [2] conducted an extensive investigation into the behaviour of particles across oblique shocks, using a definition for the drag coefficient [3] of

\[ C_D = \frac{24}{Re (1 + 2.7 Kn_e)} \]

Here \( Kn_e \) is the Knudsen number, determined by the ratio of the mean free path of the gas \( l \) to the particle diameter:

\[ Kn_e = \frac{l}{d_p} \]

Thus, they determined the particle relaxation time \( \tau_p \) to be given by the equation:

\[ \tau_p = \frac{D_m d_p^2}{18 \zeta} \]

From Urban and Mungal [2] the practical particle relaxation time for fine-grade Aluminium Oxide is 20-28 \( \mu s \), compared to the theoretical estimates of 2.3\( \mu s \). According to Yuceil et al [4], when moving across the Mach disk in the centre of the jet, the flow velocity may be reduced by as much as a factor of 3. The velocity of the particles however, will be unchanged. From 1-D isentropic nozzle calculations, the flow velocity can be estimated to be approximately 540m/s at nozzle exit, so this is taken as the particle velocity prior to the Mach disk. At this speed, the particle relaxation length can be calculated as:

\[ \zeta = 13.5 \text{mm} \]

i.e. the particle could travel 13.5mm downstream of the shock before it sufficiently matched the flow velocity according to the criterion. This is half the high resolution measurement volume, making the internal structures of the jet difficult to measure downstream of the shock This is a worst case scenario; the theoretical relaxation time for the aluminium oxide is 2.3\( \mu s \), and it is difficult to estimate the actual size of particles in the jet, since the efficiency of the seeder is unknown. Additionally, the speed of the flow steadily increases after a shock wave due to the mixing from the slip lines around the Mach disk, so even if particle performance is as poor as in prior experiments, the particle relaxation distance will be noticeably reduced compared to this estimate.

The limitations of high speed PIV are brought into sharpest relief when considering the motion of a flow through a shock. The area downstream of a shock will return incorrect velocity estimates due to particle lag, but on the other hand even if the particle responds correctly, large enough window sizes will actually impact on the velocity estimates upstream of the shock due to averaging effects [5].
2. Experimental Setup and Methodology
The flow was generated via a compressor, and seeded via passage through a cyclonic seeder as per Glass and Kennedy [6]. The nozzle was a DeLaval nozzle with a 2.5mm diameter throat, and a 2.9mm diameter exit aperture.

2.1 Optical Setup
A dual cavity pulsed Nd:YAG laser with a wavelength of 532nm provided illumination of the seed particles in the flow, firing 6ns duration pulses. A PCO Sensicam camera with a 12-bit cooled 1280 x 1024 pixel CCD array recorded the PIV image pairs with various $\Delta t$ down to 300ns. Synchronization of the laser and camera was achieved through use of in-house software on a real-time Linux system. Operating the camera at $t_{\Delta t}$ below 600ns caused the two images in each pair to bleed together at the edges, which appeared to be a limit imposed by the camera system. For future work, a dual camera setup operating through a prism could be employed to allow for much smaller $t_{\Delta t}$.

2.2 Particle Selection and Delivery System
Aluminium Oxide was chosen as the seeding powder, as it is widely used in the measurement of combusting flows and high speed flows. The particles have a nominal size of 0.3 $\mu m$ however Urban and Mungal [2] concluded after testing that agglomeration tended to produce discrete particulate structures one order of magnitude larger than the nominal particle size. The cyclonic seeder was based on the design of Glass and Kennedy [6], however very little information was available as to the design methodology for such a device. Without a nozzle attached there is sufficient volumetric flow rate to entrain a large amount of powder in the flow. However when the flow is choked, despite the fact that the mass flow rate is unchanged, the higher density results in a lower volumetric flow rate, and thus lower specific momentum of the fluid. This is presumed to be the reason that after an initial period of effective seeding caused by initial excitation of the powder, seed density in the flow drops steadily. In retrospect, the volume of the vessel may also have been too large for the flow rates involved. Further investigation is required to determine the ability of a cyclonic type seeder to seed a flow of this nature. Other potential approaches would be the use of a fluidized bed or atomizer.

2.3 MCCDPIV
The Multigrid Cross-Correlation Digital Particle Image Velocimetry (MCCDPIV) algorithm for analysing the image pairs is described in Soria [7]. It utilizes an adaptive technique to increase the velocity dynamic range while reducing the bias and random errors inherent in standard CCDPIV. The adaptive technique is based on the principle that the locations of the interrogation windows in the two images of the pair do not necessarily have to coincide. By obtaining a local velocity estimate using a large interrogation window, the second interrogation window can then be displaced by converting this velocity estimate into a displacement.

This displacement effectively increases the measurable dynamic range of velocity and reduces the uncertainty of the measurement [7]. This is advantageous when examining high speed flows, where the high particle displacements necessitate the use of larger interrogation windows. In this case only a single grid level was used.

To produce an acceptable percentage of valid vectors, pixel displacements should be maintained at less than 25% of the interrogation window size. Due to the $\Delta t$ limitation of the optical system, this meant that increasing the magnification of the optical system actually necessitated the use of larger interrogation windows, thus reducing the gain in resolution.

3. Results
The experimental conditions for the experiment are presented in Table 1 below.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Measurement Region (mm) [x/r]</th>
<th>Magnification (px/mm)</th>
<th>Vector Spacing (px)[mm]</th>
<th>Interrogation Window (px)[mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Resolution Scattering</td>
<td>75/60</td>
<td>17.07</td>
<td>(0) [0.94]</td>
<td>(32x32) [1.87x1.87]</td>
</tr>
<tr>
<td>Low Resolution PIV</td>
<td>75/60</td>
<td>17.07</td>
<td>(16) [0.94]</td>
<td>(32x32) [1.87x1.87]</td>
</tr>
<tr>
<td>High Resolution PIV</td>
<td>25/20</td>
<td>51.2</td>
<td>(32) [0.625]</td>
<td>(64x64) [1.25x1.25]</td>
</tr>
</tbody>
</table>

Table 1: Experimental Conditions
3.1 Mie Scattering Results
An instantaneous and Mie scattering image of the flow is presented in Fig 2. A classical turbulent jet structure is evident, while close examination of the nearfield area demonstrates an initial expansion immediately following the nozzle exit, characteristic of an underexpanded jet.

Figure 2: Instantaneous Mie Scattering Image

3.2 Low Resolution PIV
The mean velocity vector and contour fields for the low resolution PIV measurements are shown in Fig 3. The double hump velocity profile characteristic of an underexpanded supersonic jet is immediately apparent. The peak measured velocity was 620m/s, which corresponds to a Mach number of 1.82.

Figure 3: Low Resolution PIV Measurements:
A) Velocity Vectors (Every second vector skipped),
B) Velocity Contours

Fig 4 presents a vorticity contour for the low resolution case.

Figure 4: Low Resolution PIV Measurement Vorticity Contours

Previous experiments, such as Yuceil et al [4], were able to determine the location of the slip lines around the Mach disk by examining the vorticity of the flow, however they were unable to achieve this for a weakly underexpanded jet. Thus we are unable to determine if our resolution would be sufficient to resolve these features.

3.3 High Resolution PIV
For the high resolution case, we increased our magnification by a factor of three, but as stated in section 2, this only effectively increased our resolution by about 1.5. Examination of the instantaneous PIV images is instructive in terms of the problems due to irregular seeding density. Fig 5 presents a typical instantaneous PIV image. The location of the regions of invalid vectors varied from image to image.

Figure 5: High Resolution PIV Measurement Instantaneous Image.

The non-uniformity of seeding is one issue that must be overcome if instantaneous analysis is to be done on the jet.

Fig 6 presents the mean velocity vector and contour fields for the high resolution case.

Figure 6: High Resolution PIV Measurements:
A) Velocity Vectors (Every second vector skipped),
B) Velocity Contours

The same basic structure of lower velocity in the centerline, and expansion to higher supersonic velocities to either side of the centerline is evident. In the high resolution case the remixing between the subsonic flow in the centerline and the supersonic flow to either side is more evident. According to theory presented in Anderson [8], and some experimental findings such as [4],[9] mixing between the supersonic and subsonic sections of the flow should produce supersonic flow in the centerline some distance downstream of a Mach disk in
a weak underexpanded jet such as this one, which is apparent in these results.
The difficulty in resolving the internal structures of the flow can be highlighted by examining the size of the interrogation windows compared to the scale of structure in the flow. The distance from the jet centerline to the slip lines around the Mach disk is approximately 1.5 mm, which at this magnification is equivalent to 81 px. With an interrogation window size of 64 pixels, it is easy to see that the averaging process will make resolving accurate vectors in this region difficult.

3.4 Velocity Profiles

The normalized streamwise velocity profiles calculated for both the High and Low Resolution cases are presented in Fig 7. Directly following the nozzle exit the velocity profile takes a bell shape, however the Prandtl-Meyer expansion waves in the jet accelerate the flow towards the outer edges of the flow. A strong normal shockwave in the core of the jet results in a sharp decrease in velocity at the centre, while weaker oblique shock waves offer a smaller reduction in velocity near the edges of the jet, resulting in the double hump velocity profile that is evident 5 nozzle diameters downstream in Fig 7A. In the High Resolution case, the double hump profile may be resolved earlier due to finer resolution. In both cases, by the time the flow has traveled 8 nozzle diameters downstream, it has returned to a bell shape, as typical of subsonic flow. After ten nozzle diameters, the flow has achieved self-similarity, though the curves are omitted from this paper for the sake of brevity.

4. Conclusion

This paper has described an experiment designed to measure the velocity field of a supersonic jet via Particle Image Velocimetry. A double-hump mean streamwise velocity profile was measured similar to that demonstrated in previous literature for an underexpanded supersonic jet. The camera used was shown to be the limiting factor in attempting to minimize the pixel displacements required for the experiment. There is evidence to suggest the existence of shocks in the jet, however the inability of PIV to resolve their location has been reinforced. The ability of the Al2O3 to follow the flow is difficult to ascertain from these measurements, however a larger diameter nozzle and a more strongly underexpanded jet would likely provide a suitable base to assess this. Uniformity of seeding is shown to be crucial in obtaining valid instantaneous measurements, with the current seeding system demonstrated to be adequate for taking mean flow measurements, but lacking consistency for instantaneous measurements. With an improved seeding system that ensures adequate particle distribution, and a dual camera system to allow for lower Δt, this approach could prove an effective tool for quantitative measurement of rocket or gas turbine exhaust plumes.

5. References


